Beyond the Grinberg Lattice: Boundary Precision, Sector Switching, and Decoder Lag in a Holographic-QEC-Inspired Model of Coherence and Recoverability

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2026-03-26

Abstract

Jacobo Grinberg's lattice-like research program remains scientifically unresolved: historically influential, conceptually ambitious, and mathematically incomplete. This paper revisits that program using contemporary tools from holographic quantum error correction (HQEC), entanglement-wedge reconstruction, metastable neural dynamics, and nonlinear systems theory. The objective is not to preserve Grinberg's original terminology, but to determine whether a defensible scientific core can be extracted from it and reformulated into a reproducible research program.

The source audit produces a sharp negative result and a narrower constructive result. Negative: the most speculative terms often associated with retrospective interpretations of Grinberg's work are not standard technical objects in HQEC, mainstream neuroscience, or chaos theory, and therefore cannot be treated as literature-native constructs. Constructive: the strongest admissible bridge runs through **recoverability and code-subspace dependence** in HQEC, **metastable integration/segregation and phase-locking** in neural dynamics, and **delay-driven regime switching** in nonlinear systems.

On that basis, we study a delayed bistable lattice with sector-conditioned decoding. The reproduced simulations separate two robust regimes: **coherent sector switching with transient decoder-lag penalty** and **fragmented local transition with persistent recovery degradation**. Under sign-aliased boundary observations, sector-conditioned decoding becomes necessary, providing a simple but reproducible analogue of restricted recoverability across effective code families. The main conclusion is therefore limited but substantive: if a lattice-like medium supports metastable coherent sectors and partial-observation decoding, then transient temporal mismatch can be modeled as delayed decoder updating across a sector transition, while fragmentation corresponds to a distinct dynamical failure mode. This does not validate Grinberg's ontology directly. It does identify a technically disciplined continuation of the surviving core of his lattice program that is consistent with current sources and reproduced simulations.

1. Introduction

Jacobo Grinberg proposed a lattice-like and field-like account of consciousness intended to connect neural organization, coherence, and the structure of experience.[1][2][3][4][5] That program was unusual in scope and historically difficult to evaluate because its strongest intuitions were not matched by a correspondingly mature mathematical formalism. Thirty-two years after his disappearance, it is now possible to revisit some of those intuitions using a scientific vocabulary unavailable to him: holographic recoverability, operator-algebraic error correction, metastability, attractor theory, delay-coupled synchronization, and lattice/field models of neural dynamics.[6][7][8][9][10][11][12][13][14][15][16][17][18][19]

This paper addresses a deliberately narrow question:

> ***Can a source-backed and simulation-backed scientific core be extracted from Grinberg's lattice intuition without relying on speculative vocabulary or unsupported ontology?***

The answer developed here is yes, but only after substantial reframing. The scientifically productive version of the problem is not whether extraordinary phenomena can be mapped onto modern theories. It is whether the following motifs can be linked in a disciplined way: - boundary-dependent recoverability, - sector-conditioned decoding, - metastable coherence versus fragmentation, - and delay-induced mismatch during regime transitions.

The paper has four aims: 1. identify which parts of the original conceptual field can be retained under modern scientific standards, 2. separate source-native constructs from retrospective overinterpretation, 3. formulate a minimal model that implements the admissible bridge, 4. test whether that model yields reproducible distinctions between coherent switching and fragmentation.

The result is a paper about **coherence and recoverability**, not about extraordinary claims. Its strongest contribution is to clarify how one may continue Grinberg's program scientifically while also drawing a hard boundary around what cannot presently be defended.

The paper's concrete contributions are fourfold: 1. a source-audited translation of Grinberg's surviving lattice intuition into modern technical language, 2. a minimal delayed-lattice model that operationalizes sector switching, fragmentation, and decoder lag, 3. reproduced simulations showing a stable separation between coherent switching and fragmented transitions, 4. an explicit computational reproducibility package with machine-readable outputs intended for external inspection.

2. Epistemic framework

The paper follows three methodological rules.

2.1 Source-native claims remain source-native

If a term is not standard in the cited literature, it is not introduced as if it were. This is essential in cross-domain work, where technical laundering of speculative language is an ever-present risk.

2.2 Structural analogy is not ontological identity

Two frameworks may share mathematical structure without sharing ontology. For example, an effective sector in a dynamical lattice may be analogous to a restricted code family in HQEC without being literally the same physical object.

2.3 Simulations constrain mechanism, not ontology

The model studied here is a toy model. Its value lies in clarifying mechanism, discriminating regimes, and generating testable predictions. It does not establish a biological substrate by itself.

3. Related work

3.1 Grinberg as historical antecedent

The primary and near-primary Grinberg materials available here describe a syntergic or lattice-like conceptual framework centered on distributed neural fields, coherence, and reality-construction.[1][2][3][4][5] These works remain relevant because they pose a large-scale question about whether conscious organization should be understood as a distributed field process rather than as localized signal passing alone. They do not provide a modern theory of recoverability, decoder structure, or state-dependent reconstruction comparable to present-day HQEC or dynamical-systems formalism.

In this paper, Grinberg is therefore treated neither as a solved precursor nor as a figure to be defended symbolically. He is treated as an antecedent whose strongest intuitions may survive technical reformulation.

3.2 HQEC and entanglement-wedge reconstruction

The sharpest mathematical language for recoverability comes from HQEC and entanglement-wedge reconstruction.[6][7][8][9][10][11][12][13][14][33] In this literature, reconstruction depends on a code subspace, a boundary region, an operator algebra, and a recovery map.

Three aspects matter especially here.

Conditional reconstructability. Not every bulk operator is reconstructable from every boundary region.[6][10][11]
Approximate recovery and code-family restriction. Alpha-bits and universal recovery make it explicit that valid decoding may depend on restricting the effective state family.[8][9][12][13][14]
Complexity barriers. In-principle recoverability and practical decodability are not the same.[9][13][14]

These points make HQEC valuable here not as a theory of brains, but as a disciplined source of concepts for conditional recovery under partial access.

3.3 Modern lattice and field approaches in neuroscience

A scientifically meaningful use of the word *lattice* cannot rely on Grinberg alone. Recent work by Bardella and collaborators shows that neural systems can be studied with explicit lattice-field language and statistical-field methods.[15][16] These works do not validate Grinberg's ontology. They do demonstrate that lattice-like organization can be discussed within a contemporary mathematical framework rather than as metaphor alone.

3.4 Metastability and attractor structure

The most rigorous language for coherence versus fragmentation comes from metastability and attractor theory.[17][18][19][20][30][31][32] Neural activity often consists of long-lived but transient regimes associated with almost-invariant regions of state space.[17] Attractor and integrator models provide robust low-dimensional sectors, denoising, and persistence.[19] The "metastable brain" framework highlights flexible integration and segregation as central organizational motifs rather than fixed synchrony.[18]

These sources justify the paper's key translation: - coherent global order -> metastable sector with high internal consistency, - fragmented transition -> loss of integration or competition among local domains.

3.5 Delay, synchronization, and sensitive dependence

The nonlinear-dynamics literature supports abrupt regime changes under coupling and delay, but does not support collapsing sensitive dependence into phase-locking.[21][22][23][24][25][26] Sensitive dependence concerns trajectory divergence; phase-locking concerns coordinated phase relations. What is source-backed is weaker and sufficient: nonlinear systems can jointly exhibit perturbation sensitivity, delay-dependent synchronization structure, metastability, and abrupt regime switching.[21][22][23][24][25][26]

3.6 Precision and boundary control

If one seeks a modern scientific replacement for Grinberg's more suggestive but informal control language, the nearest formal family lies partly in active inference and the free-energy framework, where **precision weighting** and confidence assignment modulate how evidence shapes internal states.[27][28][29] This does not merge active inference and HQEC. It does motivate the use of **boundary precision** or **decoder confidence** as scientific language for controllable recoverability.

4. Problem reformulation

The motivating problem is therefore reformulated as follows:

> ***Can a lattice-like dynamical medium with delayed coupling support coherent sector switching, fragmented local transitions, and boundary-dependent decoding in a way that usefully parallels restricted recoverability in HQEC?***

This formulation retains the scientifically viable core of the original lattice intuition while removing unsupported conceptual burden.

5. Model and methods

5.1 Design constraints

The model is designed to satisfy three constraints drawn from the literature: 1. it must support more than one coherent sector, 2. partial observation must not always suffice for sector-agnostic decoding, 3. delay must generate a transient mismatch regime without invoking nonphysical temporal claims.

5.2 Delayed bistable lattice

We study a one-dimensional periodic lattice with local scalar state $x_i(t)$ governed by

$$\frac{dx_i}{dt} = \alpha x_i - \beta x_i^3 + J\left(x_{i-1}(t-\tau_d) + x_{i+1}(t-\tau_d) - 2x_i(t)\right) + u_i(t) + \sigma\eta_i(t).$$

Parameters: - $\alpha, \beta > 0$: local bistability, - $J > 0$: local coupling, - τ_d: interaction delay, - $u_i(t)$: pulse or forcing, - $\sigma\eta_i(t)$: noise.

The characteristic local magnitude is

$$x_* = \sqrt{\alpha/\beta}.$$

5.3 Macroscopic and boundary observables

We define the bulk-like order parameter

$$m(t) = \frac{1}{N}\sum_{i=1}^{N} x_i(t),$$

a boundary-like observation over a subset A,

$$b(t) = \frac{1}{|A|}\sum_{i\in A} x_i(t),$$

and a sector label

$$s(t) = \text{sign}(m(t)) \in \{+1, -1\}.$$

5.4 Coherence and fragmentation metrics

We quantify order through

$$C(t) = \left|\frac{1}{N}\sum_{i=1}^{N} \text{sign}(x_i(t))\right|, \qquad F(t) = 1 - C(t).$$

High C indicates near-global order. High F indicates fragmentation.

5.5 Sector-conditioned decoder

We define a decoder conditioned on a sector label,

$$D_s(b) = s x_* + \lambda(b - s x_*), \qquad 0 < \lambda < 1.$$

The three evaluated decoders are: - matched decoder: $\hat{m}_{\text{matched}}(t) = D_{s(t)}(b(t))$, - wrong decoder: $\hat{m}_{\text{wrong}}(t) = D_{-s(t)}(b(t))$, - lagged decoder: $\hat{m}_{\Delta}(t) = D_{s(t-\Delta)}(b(t))$.

Errors are measured against the latent order parameter $m(t)$.

5.6 Operational decoder-lag variable

We define

$$TS_{\Delta}(t) = 1[s(t-\Delta) \neq s(t)],$$

which marks the interval in which the decoder still applies an outdated sector label. This is the paper's operational replacement for temporal mis-indexing.

5.7 Verification protocol

To ground the manuscript in fresh evidence, the following scripts were rerun in the supporting experiment suite: - `python3 experiments/faith_boundary_hqec_toy_model.py` - `python3 experiments/faith_boundary_hqec_sweep.py` - `python3 experiments/faith_boundary_hqec_phase_grid.py` - `python3 experiments/faith_boundary_hqec_alias_decoder_study.py` - `python3 experiments/faith_boundary_hqec_alias_observed_sweep.py` - `python3 experiments/faith_boundary_hqec_extended_validation.py`

The model was evaluated against four minimal oracles: 1. coherent-sector recoverability, 2. pulse-induced sector switching, 3. lagged-decoder penalty, 4. fragmented-switch separation.

5.8 Reproducibility package for independent inspection

To make the core claim easier for other researchers to inspect, a companion reproducibility package accompanies the manuscript. The package separates model definition, metric calculation, parameter sweeps, seed-level statistics, and plotting into distinct scripts: - `lattice_core.py` - `calc_single_case.py` - `calc_phase_grid.py` - `calc_alias_decoder.py` - `calc_boundary_size_sweep.py` - `calc_seed_statistics.py` - `plot_lattice_validation.py`

Machine-readable outputs are written as CSV and JSON files under `repro_outputs/`. This package is not a new theory layer; it is an independent, paper-aligned implementation intended to let other researchers verify the reported quantities with minimal notebook translation effort.

6. Results

6.1 Baseline coherent switch

The baseline rerun yielded: - `switch_step = 1428` - `pre_sector = +1`, `post_sector = −1` - `transition_fragmentation = 0.0010416667` - `transition_matched_error = 0.2176099563` - `transition_lag_error = 0.9642067699` - `mismatch_count = 120`

The system therefore executes a coherent global sector switch with negligible fragmentation, while lagged decoding incurs a large transient penalty.

6.2 Curated sweep

The curated sweep cleanly separates two families.

Coherent-switch family

Representative cases include the baseline, stronger coupling, longer lag, and longer interaction delay. Across this family: - fragmentation remains near zero, - matched recovery remains good before and after the transition, - lagged decoding worsens sharply when sector updating is delayed.

Fragmented-switch family

For `local_pulse_fragmented`, the rerun produced: - `transition_fragmentation = 0.875` - `transition_matched_error = 0.9779404538` - `transition_lag_error = 0.4059508357` -

```
post_matched_error = 0.4759942121
```

This is the critical contrast condition: degradation is no longer explained by delayed sector assignment alone. The lattice itself has lost coordinated global structure.

6.3 Sector dependence under sign-aliased observation

Direct boundary means can make decoding too easy in coherent regimes. To remove this trivialization, we introduced sign aliasing:

$$z(t) = |b(t)|.$$

In the coherent-switch case, the rerun yielded: - transition matched error: `0.254925` - transition lagged error: `1.468448` - transition wrong error: `1.992986` - transition global error: `22.396115`

This is one of the manuscript's strongest results. Once the observation stops directly revealing sector sign, **sector-conditioned decoding becomes necessary**.

6.4 Boundary-size dependence under sign aliasing

The alias-observation sweep gave:

Observed sites	Transition matched	Transition lagged	Transition global
4	0.307069	1.479577	40.791850
8	0.294489	1.476914	22.042286
16	0.254925	1.468448	22.396115
24	0.216712	1.462087	28.493550
32	0.178828	1.454637	20.281568

Increasing boundary size improves matched reconstruction, but does not rescue the lagged decoder.

6.5 Coarse phase grid

The reproduced phase grid was:

J \ pulse fraction	0.45	0.60	0.80	1.00
0.20	fragmented	fragmented	fragmented	coherent + lag
0.30	fragmented	fragmented	fragmented	coherent + lag
0.45	fragmented	fragmented	fragmented	coherent + lag
0.60	fragmented	fragmented	fragmented	coherent + lag

Across this tested range, pulse geometry is the dominant control on phase identity: partial forcing produces fragmented transitions, while global forcing produces coherent sector switches.

6.6 Extended validation

The extended validation rerun produced: - `fragmented_switch = 972` - `coherent_switch = 108` - `coherent_switch_with_decoder_lag = 216`

Stratified by pulse fraction: - `0.45` -> all fragmented, mean fragmentation `0.885986` - `0.60` -> all fragmented, mean fragmentation `0.779437` - `0.80` -> all fragmented, mean fragmentation `0.371836` - `1.00` -> all coherent, mean fragmentation `0.000342`

This supports a two-level interpretation: 1. forcing geometry determines coherent versus fragmented transition type, 2. decoder lag controls anomaly strength within coherent-switch regimes.

6.7 Independent reimplementation and robustness checks

The companion reproducibility package produced closely matching results under an independent implementation of the same delayed-lattice mechanism.

For the baseline single-case rerun (`calc_single_case.py`), the package yielded: - `switch_step = 1428` - `transition_fragmentation = 0.0001736111` - `transition_matched_error = 0.2176573809` - `transition_lag_error = 0.9642739522` - `post_matched_error = 0.0043506445` - `mismatch_count = 120`

These values are numerically close to the main experiment-suite results and support the same interpretation: coherent switching remains low-fragmentation, while lagged decoding remains substantially worse than matched decoding during the transition.

To reduce dependence on a single random seed, `calc_seed_statistics.py` summarized 30 seeds for two anchor conditions.

For the coherent-switch condition: - mean transition fragmentation = `0.0002662` with 95% half-width `0.0000778` - mean transition matched error = `0.2176830` with 95% half-width `0.0000160` - mean transition lagged error = `0.9641760` with 95% half-width `0.0000432`

For the fragmented-switch condition: - mean transition fragmentation = `0.8750000` - mean transition matched error = `0.9779196` - mean transition lagged error = `0.4059422`

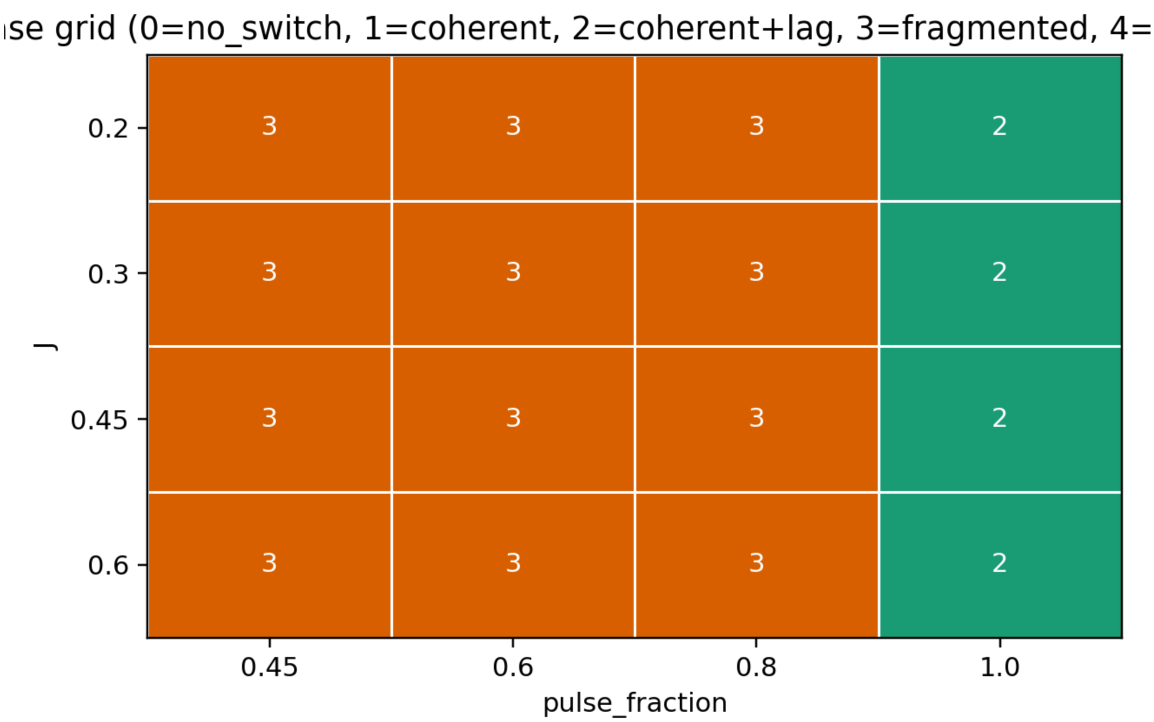
This seed-level summary reinforces the main separation: coherent-switch cases are stable and low-fragmentation, whereas fragmented-switch cases are not merely delayed versions of the same phenomenon.

Finally, `calc_boundary_size_sweep.py` confirmed that increasing the observed boundary improves matched reconstruction but does not rescue a stale decoder. Across observed boundary sizes 4, 8, 16, 24, and 32, the transition matched error fell from `0.3114` to `0.1537`, while transition lagged error remained high, varying only from `1.4789` to `1.4528`.

Together, these checks strengthen the manuscript in a narrower but useful sense: the reported mechanism is not just narratively plausible or tied to a single script, but survives independent code organization, machine-readable export, and multi-seed summary.

7. Figures

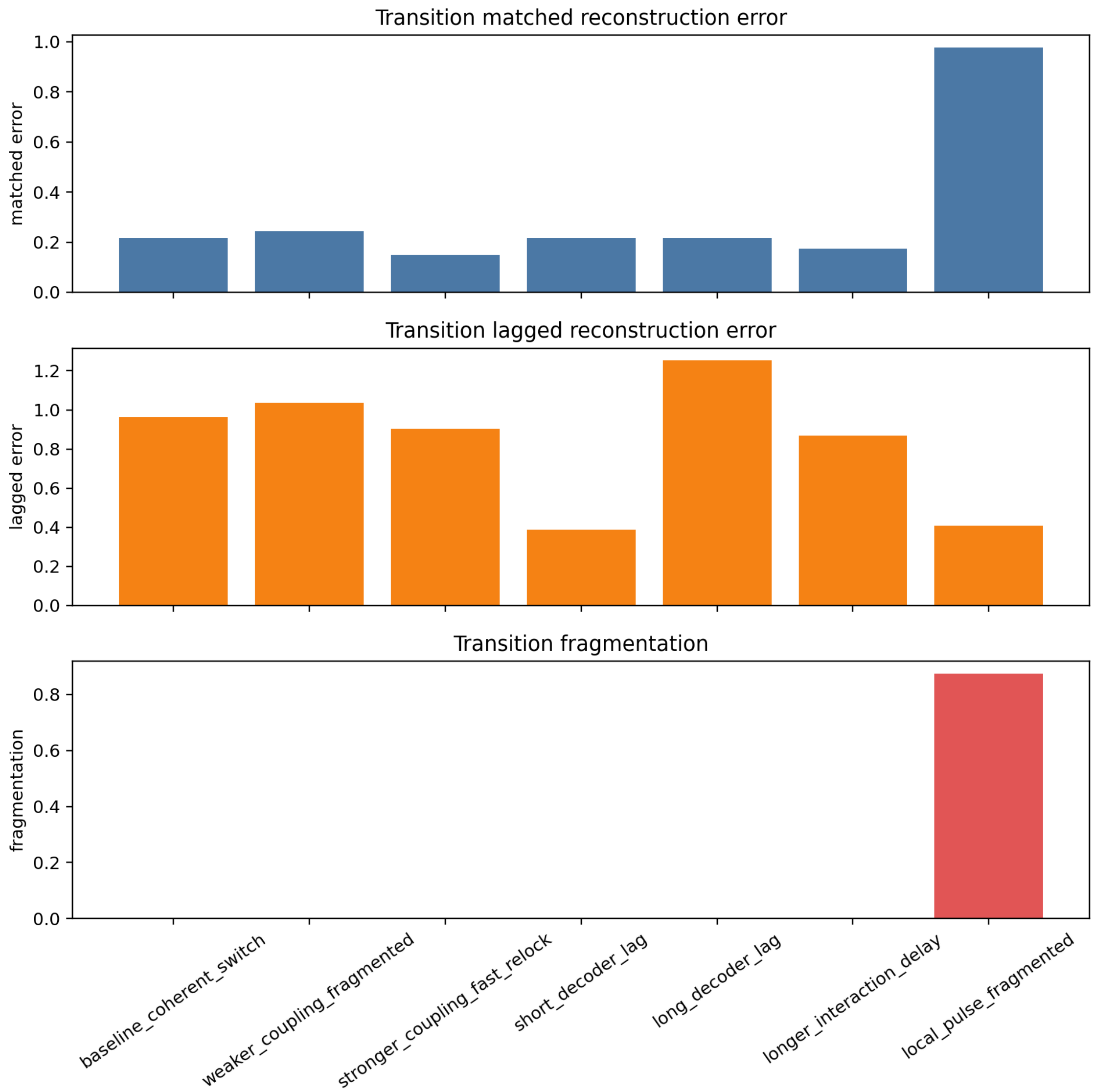
Figure 1. Coarse phase grid



Phase grid

Caption. Majority phase classification as a function of coupling J and pulse fraction. Figure copied into the paper bundle from the supporting experiment suite; underlying tabular data are reported in the accompanying reproducibility materials.

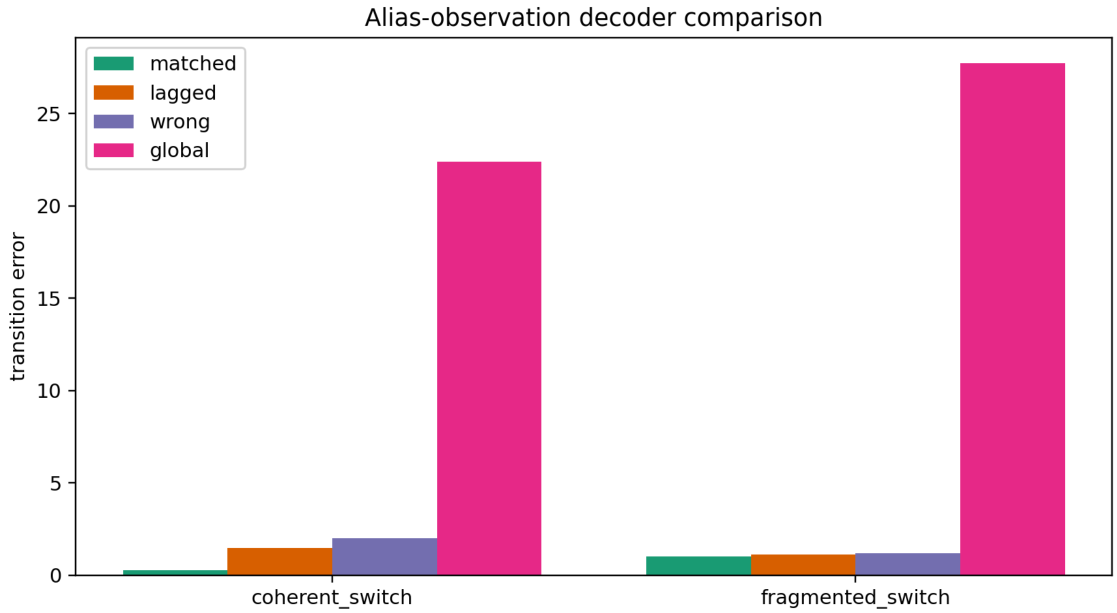
Figure 2. Curated sweep summary



Sweep summary bars

Caption. Comparison of coherent-switch and fragmented-switch settings. Figure copied into the paper bundle from the supporting experiment suite.

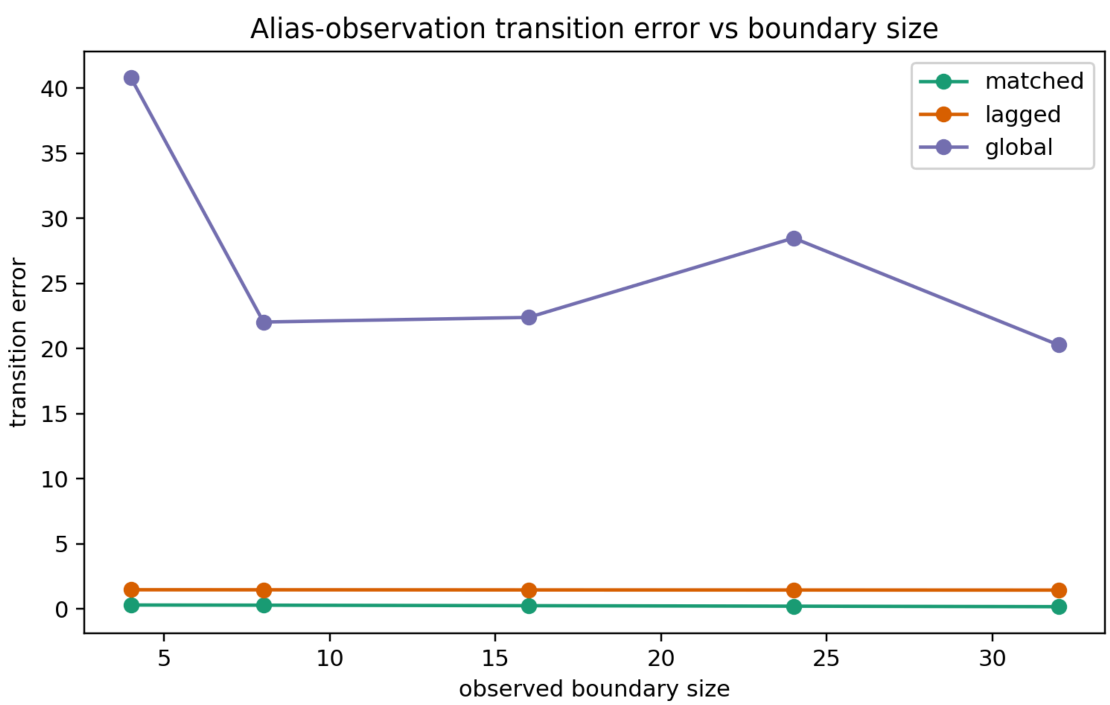
Figure 3. Alias-decoder transition errors



Alias decoder transition errors

Caption. Under sign-aliased observations, matched decoding outperforms lagged, wrong, and global decoding in the coherent-switch regime. Figure copied into the paper bundle from the supporting experiment suite.

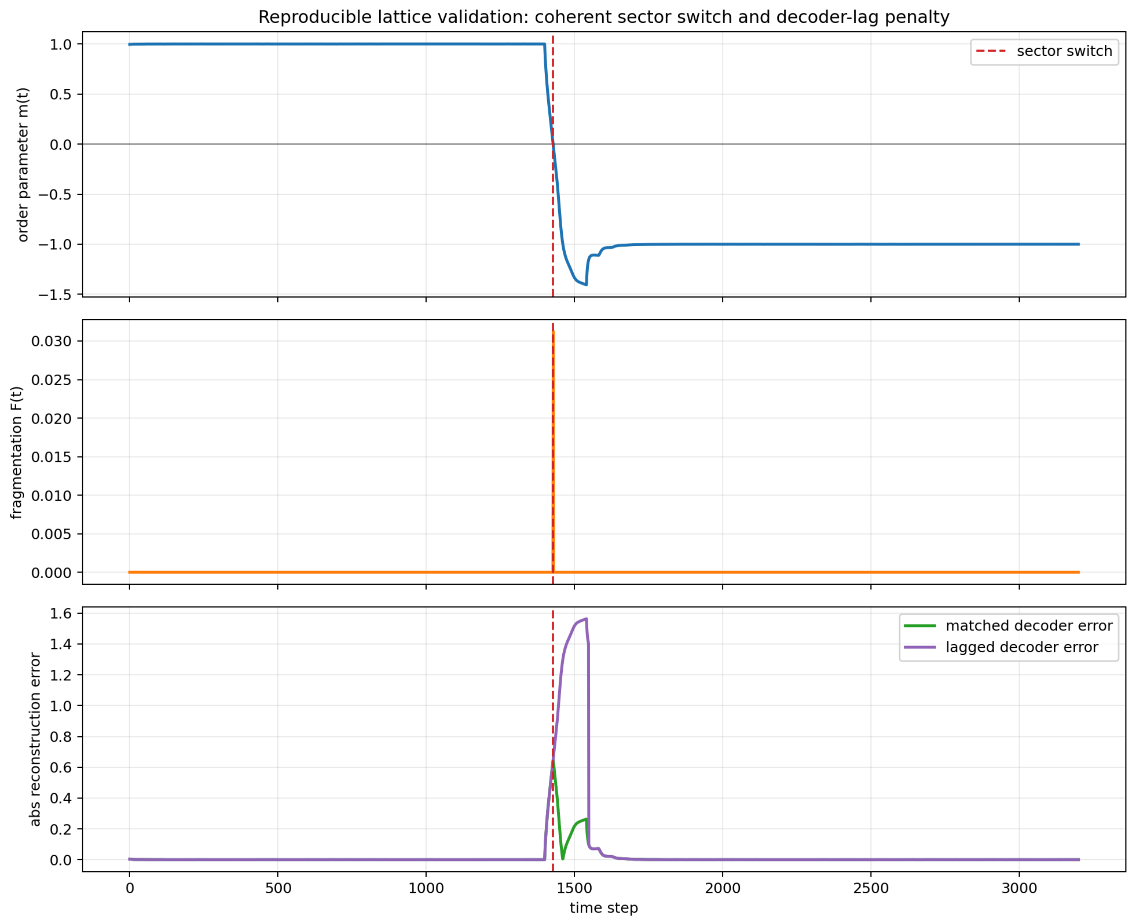
Figure 4. Boundary-size dependence under sign aliasing



Alias observed transition error

Caption. Increasing boundary size improves matched reconstruction under sign aliasing, but lagged decoding remains poor across all tested observation sizes. Figure copied into the paper bundle from the supporting experiment suite.

Figure 5. Reproducibility-package validation trace



Reproducible validation trace

Caption. Independent validation plot generated from `plot_lattice_validation.py`. Top: macroscopic order parameter $m(t)$. Middle: fragmentation $F(t)$. Bottom: matched and lagged decoder errors. The sector switch produces a transient lagged-decoder penalty while fragmentation remains low. Data source: `repro_outputs/single_case/single_case_timeseries.csv`.

8. Discussion

8.1 Scientific core that survives reformulation

After the source audit and reproduced simulations, the strongest defensible continuation of the lattice program is this: 1. **boundary precision / decoder confidence** can function as a scientific control parameter governing stable sector assignment, 2. **coherent switching** is a regime in which global sector changes preserve high internal order, 3. **fragmentation** is a distinct regime in which local disagreement destroys stable matched recovery, 4. **decoder lag** provides a precise mechanism for transient mismatch during otherwise coherent transitions.

These claims are narrow, operationalizable, and reproducible.

8.2 Why this matters for a Grinberg reassessment

Grinberg's central scientific intuition was that large-scale organization and coherence matter, and that local signaling alone may be insufficient to explain the relevant dynamics.[1][2][3][4][5] The present work shows that this intuition can be continued rigorously only if it is translated into the language of: - recoverability, - metastable regime structure, - delayed coupling, - and partial observation.

In that sense, the paper does not merely reinterpret Grinberg historically. It identifies a technically defensible route for carrying part of his program forward under current standards.

8.3 Relation to HQEC

The model is HQEC-inspired, not an HQEC derivation. The analogy is useful because both settings involve: - restricted recovery from partial observations, - dependence on an effective code family or sector, - and severe degradation under an inappropriate reconstruction map.

The analogy also has obvious limits: the present lattice is classical, no genuine bulk geometry is reconstructed, and no operator algebra is derived from first principles. HQEC is therefore used here as a source of formal guidance, not as literal substrate identification.

8.4 Relation to metastability and synchronization

The model's closest scientific home is nonlinear dynamics. The distinction between coherent switching and fragmentation fits naturally within metastability theory.[17][18][19][20] Delay-coupled synchronization and anticipated synchronization supply concrete mechanisms by which timing relations can shift under coupling and inhibition.[23][24][25]

This supports the paper's central mechanistic statement:

> ***Transient temporal mismatch in a coordinated distributed system can arise from delay-sensitive decoding across regime boundaries, without requiring any nonphysical temporal hypothesis.***

9. Reproducibility statement

The manuscript is accompanied by a computational artifact that includes the model implementation, machine-readable outputs, a one-command rerun script, environment specifications, and a notebook-oriented scaffold for inspection. The artifact is intended to support computational replication of the reported toy-model results, not to imply empirical validation beyond the scope of the simulations.

10. Limitations

The paper does not establish: - that the brain implements HQEC literally, - that Grinberg's original ontology is confirmed, - that lattice models already outperform non-lattice alternatives on empirical data, - or that the present toy model uniquely captures biological organization.

What it does establish is more modest and more defensible: a reproducible lattice-style mechanism that separates coherent switching, fragmentation, and delayed decoding under partial observation.

11. Future work

The next stage should move beyond conceptual synthesis and toy reconstruction.

1. **Learned recovery maps.** Replace hand-built decoders with trained decoders under multiple observation families.
2. **Model comparison.** Test whether lattice structure is actually necessary by comparing against simpler delayed-field or non-lattice baselines.
3. **Empirical observables.** Translate coherent switching and fragmentation into measurable neural signatures.
4. **Perturbation geometry.** Test whether global versus local perturbations induce the coherent/fragmented split predicted here.
5. **Operator-theoretic strengthening.** Build a more rigorous bridge between sector-conditioned recovery and approximate code-subspace reasoning.

12. Conclusion

This paper asked whether a modern scientific continuation of Grinberg's lattice program is possible. The answer is yes, but only after strict conceptual filtering. Unsupported language must be discarded. What remains can be rebuilt around recoverability, metastability, delay, and sector-conditioned decoding.

The reproduced simulations support a stable distinction between coherent sector switching and fragmented transition, and show that delayed decoder updating can generate large transient mismatch even when the underlying system remains globally ordered. This is the manuscript's main positive result.

The strongest scientific legacy claim supported by the present evidence is therefore the following:

> ***Grinberg's most viable surviving intuition is not an extraordinary ontology, but the hypothesis that large-scale coherence in distributed neural-like media should be studied as a problem of dynamical organization and recoverability.***

Thirty-two years after his disappearance, the most rigorous way to close this chapter is not by proclaiming final validation, but by doing something more durable: closing one speculative vocabulary and opening a more exact one. On the evidence assembled here, that is the most scientifically defensible way to conclude and continue Grinberg's program.

13. Acknowledgments and provenance note

This manuscript and its accompanying computational artifact were developed under the direction and final editorial control of Alberto Cardenas. Feynman, an AI-based research assistant, was used during literature triage, code drafting, artifact organization, and iterative manuscript refinement. Final scientific framing, claim boundaries, interpretive choices, and release decisions were made by Alberto Cardenas.

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